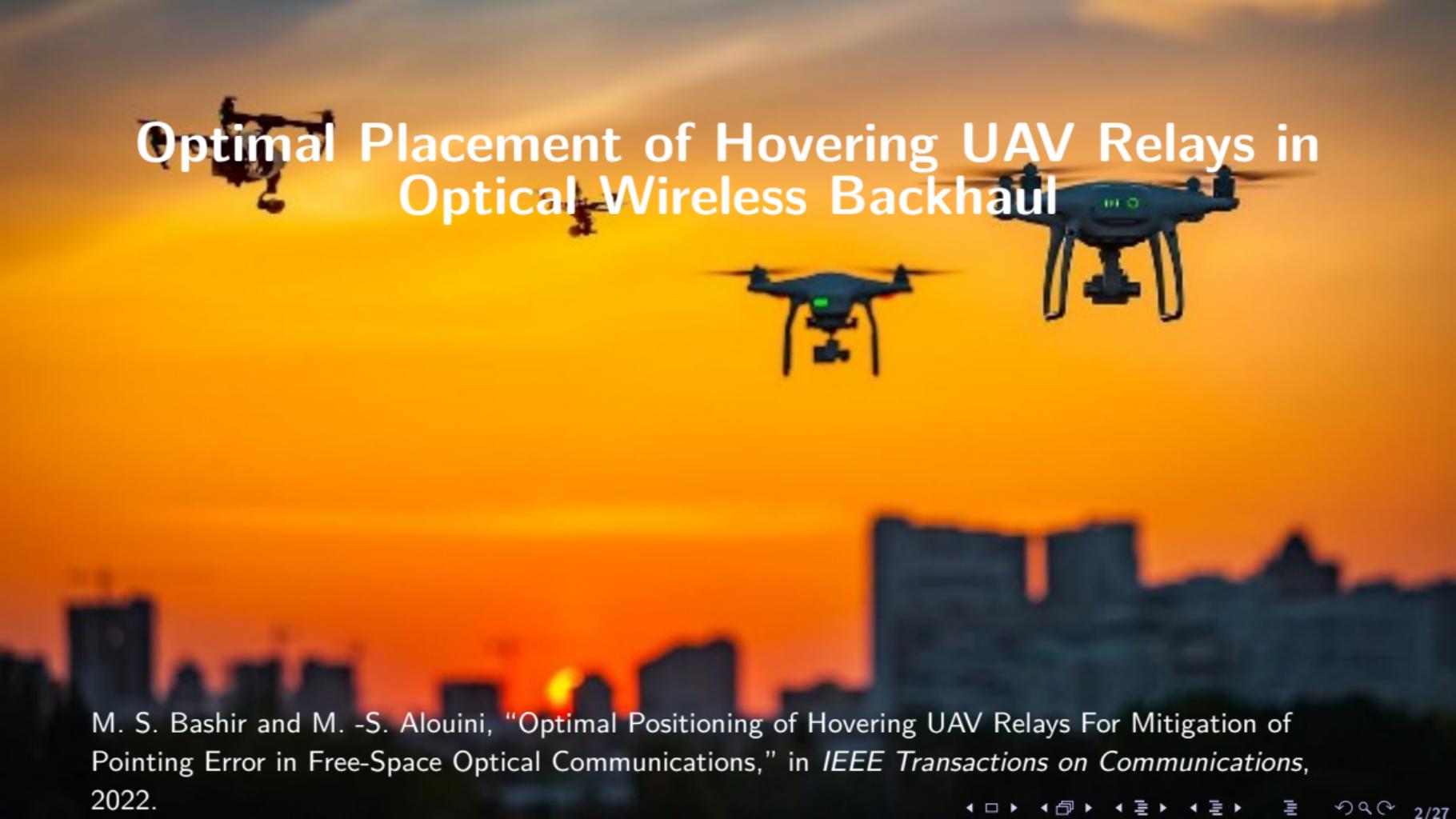


Free-Space Optical Communications With Unmanned Aerial Vehicles

Muhammad Salman Bashir

Communication Theory Lab (CTL)
King Abdullah University of Science and Technology (KAUST)

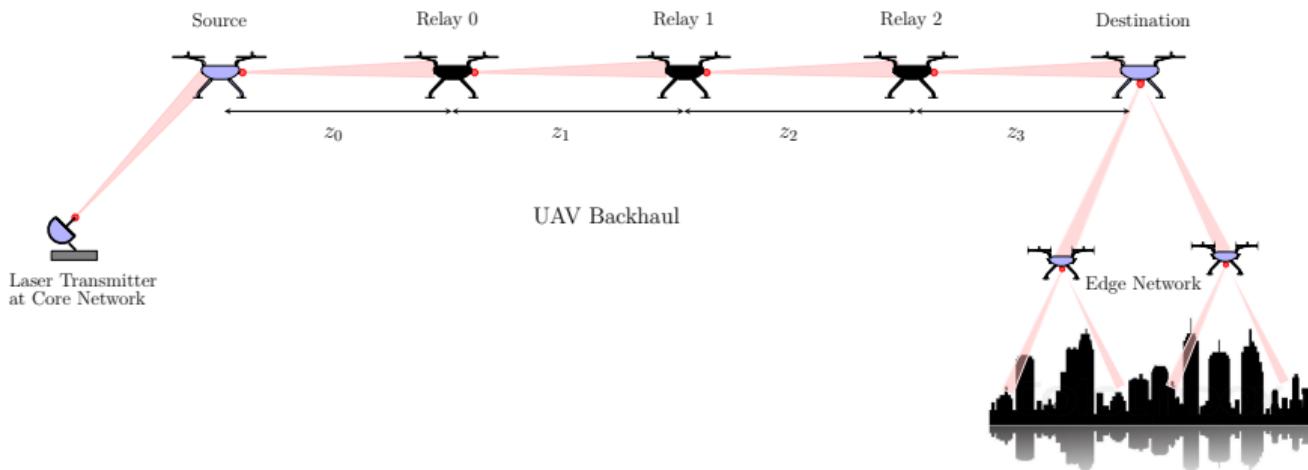


Optimal Placement of Hovering UAV Relays in Optical Wireless Backhaul

M. S. Bashir and M. -S. Alouini, "Optimal Positioning of Hovering UAV Relays For Mitigation of Pointing Error in Free-Space Optical Communications," in *IEEE Transactions on Communications*, 2022.

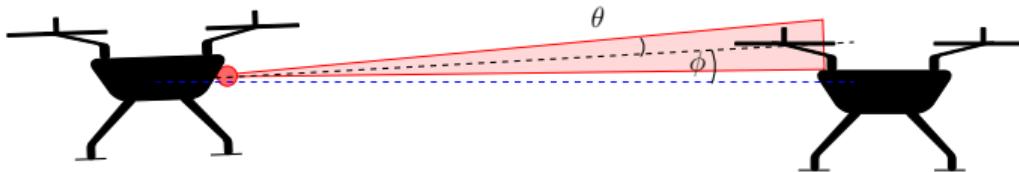
UAV Relays in Optical Wireless Backhaul

- ▶ For traditional FSO systems, the set of tunable parameters are beamwidth, receiver field-of-view, detector size, power split factor etc.
- ▶ Aerial platforms provide more degrees of freedom in terms of their flexible positions.



How do we choose distances z_0, z_1 and z_2 to minimize the outage probability?

- ▶ Pointing error



- ▶ θ : (angular) beamwidth, ϕ : (angular) pointing error
- ▶ σ : pointing error standard deviation
- ▶ Pointing error: Rayleigh distribution

$$f_h(h) = \Phi h^{(\theta^2 - \sigma^2)/\sigma^2} \cdot \mathbf{1}_{[0,B)}(h).$$

$$B := \frac{e^{-\alpha z} a^2}{2\theta^2 z^2}.$$

Pointing Error: Amplify-and-Forward Relays

- ▶ Mean channel gain: $h(z) := \mathbb{E}[\mathbf{h}(z)] = \frac{a^2}{2z^2(\theta^2 + \sigma^2)} e^{-\alpha z}$.

- ▶ CSI-assisted relays:

$$\begin{aligned} G_m &= \frac{1}{h(z_m)} = \frac{2z_m^2 (\theta^2 + \sigma^2)}{a^2} e^{\alpha z_m} \leq \mathcal{G}_u \\ &\implies z_m^2 e^{\alpha z_m} \leq \frac{\mathcal{G}_u a^2}{2\theta^2 \left(1 + \frac{\sigma^2}{\theta^2}\right)}, \quad 0 \leq m \leq M-2. \end{aligned}$$

- ▶ $\mathbb{P}(\mathcal{O}) = \mathbb{P}(\{\text{SNR} < \Upsilon_{\text{th}}\})$.

$$\mathbb{P}(\mathcal{O}) = \mathbb{P} \left(\left\{ \sum_{m=0}^{M-1} \mathcal{K}_m \exp \left(\sum_{i=0}^m \left(\frac{R_i^2}{\theta^2 z_i^2} + 2\alpha z_i \right) \right) \geq \frac{P_t^2}{\Upsilon_{\text{th}} \sigma_n^2 \sum_{m=0}^{M-1} K_m} \right\} \right).$$

- ▶ $\mathcal{K}_m := \frac{K_m}{\sum_{m=0}^{M-1} K_m}$, $K_m := \frac{4^m}{a^{4(m+1)}} \prod_{k=0}^{m-1} G_k^{-2} \prod_{j=0}^m \theta^4 z_j^4$

- ▶ $\frac{R_i^2}{\theta^2 z_i^2}$ is exponentially distributed with parameter $\lambda_i := \frac{w^2(z_i)}{2\sigma_{R_i}^2} = \frac{\theta^2}{2\sigma^2}$.

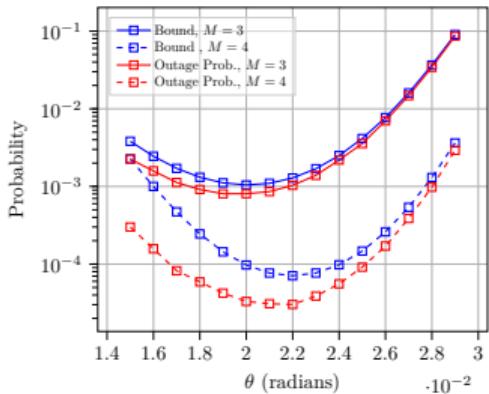
Upper Bound on Outage Probability



$$\mathbb{P}_u(\mathcal{O}) = \exp\left(-\frac{\Upsilon'_{\text{th}}}{2\frac{\sigma^2}{\theta^2}}\right) \sum_{n=0}^{M-1} \frac{1}{n!} \left(\frac{\Upsilon'_{\text{th}}}{2\frac{\sigma^2}{\theta^2}}\right)^n \cdot \mathbf{1}_{[0,\infty)}(\Upsilon'_{\text{th}}) \quad (1)$$



$$\Upsilon'_{\text{th}} = \ln\left(\frac{P_t^2 a^4 \left(1 + \frac{\sigma^2}{\theta^2}\right)^{2(M-1)}}{\Upsilon_{\text{th}} \theta^4 \sigma_n^2 \sum_{m=0}^{M-1} z_m^4 e^{2\alpha z_m} \left(1 + \frac{\sigma^2}{\theta^2}\right)^{2(M-(m+1)}}}\right) \quad (2)$$



This figure shows the outage probability and the upper bound as a function for beamwidth θ .

Optimization Problem

► Optimization Without Gain Constraint:

$$\begin{aligned}
 & \underset{x_0, \dots, x_{M-1}}{\text{minimize}} && \sum_{m=0}^{M-1} z_m^4 e^{2\alpha z_m} \left(1 + \frac{\sigma^2}{\theta^2}\right)^{2(M-(m+1))} \\
 & \text{subject to} && i) \sum_{m=0}^{M-1} z_m = D, \\
 & && ii) z_m \geq 0, \quad 0 \leq m \leq M-1.
 \end{aligned} \tag{3}$$

► The Lagrangian function

$$\mathcal{L}(x, \lambda) := \sum_{m=0}^{M-1} (z_m)^4 e^{2\alpha z_m} \left(1 + \frac{\sigma^2}{\theta^2}\right)^{2(M-(m+1))} - \lambda \left(\sum_{m=0}^{M-1} z_m - D\right). \tag{4}$$

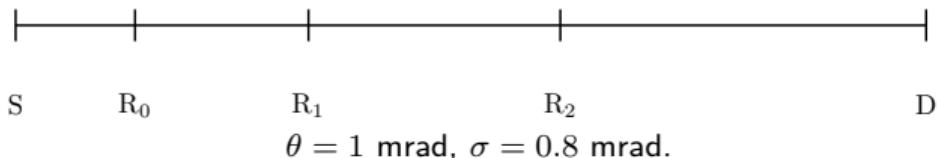
One Optimization Problem

- ▶ $z_1^* = z_0^* \left(1 + \frac{\sigma^2}{\theta^2}\right)^{\frac{2}{3}}, \quad z_2^* = z_0^* \left(1 + \frac{\sigma^2}{\theta^2}\right)^{\frac{4}{3}}, \dots, z_{M-1}^* = z_0^* \left(1 + \frac{\sigma^2}{\theta^2}\right)^{\frac{2(M-1)}{3}}$
- ▶ The general expression of optimal relay locations is

$$z_m^* = D \left(\frac{1 - (1 + \frac{\sigma^2}{\theta^2})^{\frac{2}{3}}}{1 - (1 + \frac{\sigma^2}{\theta^2})^{\frac{2M}{3}}} \right) \left(1 + \frac{\sigma^2}{\theta^2}\right)^{\frac{2m}{3}}, \quad 0 \leq m \leq M-1. \quad (5)$$

- ▶ $\frac{\sigma^2}{\theta^2}$ determines the optimal location factor $z_0^*, z_1^*, \dots, z_{M-1}^*$.
- ▶ Optimal distances form a strictly increasing sequence:

$$z_0^* < z_1^* < \dots < z_{M-1}^* \quad (6)$$



Optimization With Finite Gain Constraint \mathcal{G}_u

►

$$\begin{aligned} & \underset{x_0, \dots, x_{M-1}}{\text{minimize}} \quad \sum_{m=0}^{M-1} z_m^4 e^{2\alpha z_m} \left(1 + \frac{\sigma^2}{\theta^2}\right)^{2(M-(m+1))} \\ & \text{subject to} \quad i) \sum_{m=0}^{M-1} z_m = D, \\ & \quad ii) z_m \geq 0, \quad 0 \leq m \leq M-1, \\ & \quad iii) z_m^2 e^{\alpha z_m} \leq \frac{\mathcal{G}_u a^2}{2\theta^2 \left(1 + \frac{\sigma^2}{\theta^2}\right)}, \quad 0 \leq m \leq M-2. \end{aligned} \tag{7}$$

- For the gain constraint, there is only one point that lies in B , which is the optimal point. This optimal point satisfies $z_0^* = z_1^* = \dots = z_{M-2}^*$ and $z_{M-1}^* = D - \sum_{m=0}^{M-2} z_m^*$.

One Optimization Problem

- When $\alpha \approx 0$, the gain constraint is active when $\mathcal{G}_u \leq \frac{2\theta^2(1+\frac{\sigma^2}{\theta^2})}{a^2} (z_{M-2}^*)^2$, where

$z_{M-2}^* := D \left(\frac{1-(1+\frac{\sigma^2}{\theta^2})^{\frac{2}{3}}}{1-(1+\frac{\sigma^2}{\theta^2})^{\frac{2M}{3}}} \right) (1 + \frac{\sigma^2}{\theta^2})^{\frac{2(M-2)}{3}}$. When the constraint is active, the optimal point is

$$z_m^* = \sqrt{\frac{\mathcal{G}_u a^2}{2\theta^2 \left(1 + \frac{\sigma^2}{\theta^2}\right)}}, \quad 0 \leq m \leq M-2. \quad (8)$$

$$z_{M-1}^* = D - (M-1) \sqrt{\frac{\mathcal{G}_u a^2}{2\theta^2 \left(1 + \frac{\sigma^2}{\theta^2}\right)}}. \quad (9)$$

- Here, we have a nondecreasing sequence of link distances:

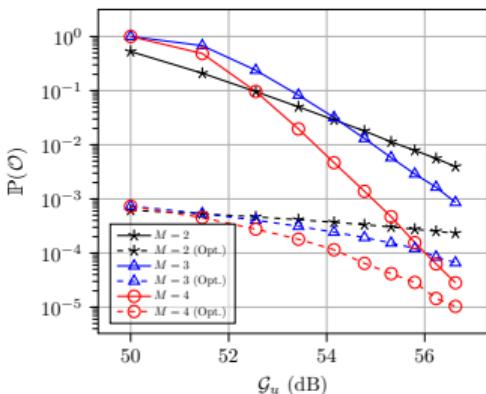
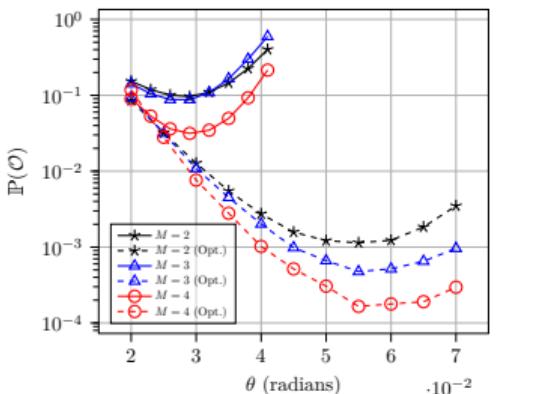
$$z_0^* = z_1^* = \cdots = z_{M-2}^* < z_{M-1}^*.$$



This figure shows the unconstrained (top) and the constrained (bottom) optimization results for $\mathcal{G}_u = 10^4$, beamwidth $\theta = 1$ mrad, and pointing error standard deviation $\sigma = 0.8$ mrad.

Numerical Results

beamwidth θ	40 mrad
pointing error standard deviation σ	20 mrad
transmit power P_t	10 Watts
maximum relay gain \mathcal{G}_u	50 dB
aperture radius a	20 cm
total link distance D	10 km
thermal noise power σ_n^2	10^{-13} Watts



Probability of outage as a function of beamwidth and gain.

Energy Optimization of Laser-Powered Hovering-UAV Relays in Optical Wireless Backhaul

M. S. Bashir and M. -S. Alouini, "Energy Optimization of a Laser-Powered Hovering-UAV Relay in Optical Wireless Backhaul," to appear in *IEEE Transactions on Wireless Communications*, 2022.

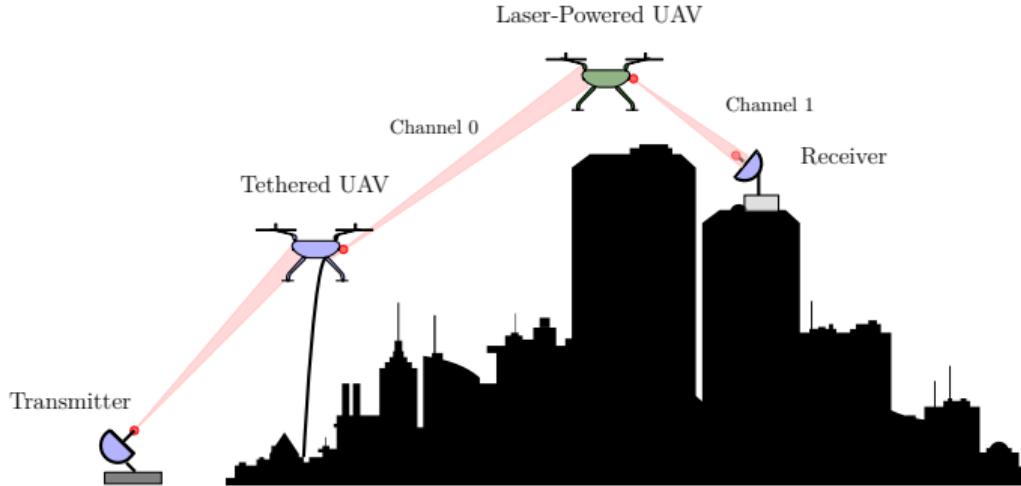
Laser-Powered UAVs: PowerLight Technologies



Courtesy: Lasermotive Inc.

- ▶ A complete transportable power beaming system which transmitted 400 watts to a mobile aerial platform over a range of 1 kilometer (2019) [PowerLight].
- ▶ Future prototype is likely to transmit 1,000 watts of power over a distance of a kilometer [PowerLight].

Laser-Powered Decode-and-Forward Relay



A laser-powered UAV is deployed to connect the source (tethered UAV) with the destination receiver in a dense urban environment.

- ▶ Channel 0 is source-relay channel; Channel 1 is relay-destination channel.
- ▶ Total end-to-end link distance is D ; z is the length of source-relay channel; $D - z$ is the length of relay-destination channel.

Channel Model

- ▶ θ is angular beamwidth and σ^2 is angular pointing error variance.
- ▶ The channel model is characterized by pointing error induced by the hover:

$$f_{\mathbb{h}_1}(h) = \Phi_1 h^{\gamma_1 - 1} \cdot \mathbb{1}_{[0, B_1)}(h), \quad (10)$$

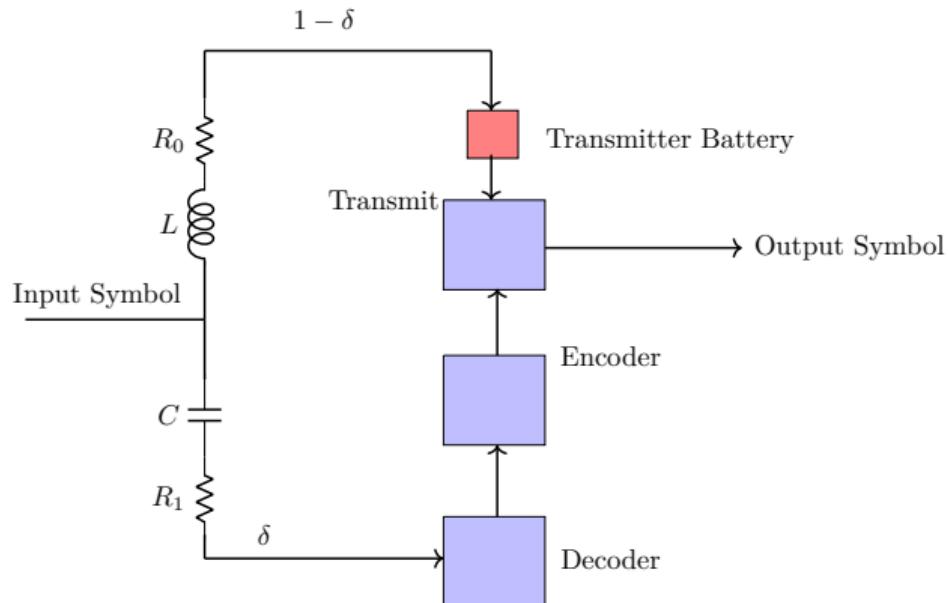
where $\Phi_1 := \gamma_1 \left(\frac{1}{B_1}\right)^{\gamma_1}$, $B_1 := \frac{e^{-\alpha(D-z)} a_d^2}{2\theta_1^2(D-z)^2}$ and $\gamma_1 := \frac{\theta_1^2}{\sigma_1^2}$.

$$f_{\mathbb{h}_0}(h) = \Phi_0 h^{\gamma_0 - 1} \cdot \mathbb{1}_{[0, B_0)}(h), \quad (11)$$

where $\Phi_0 := \gamma_0 \left(\frac{1}{B_0}\right)^{\gamma_0}$, $B_0 := \frac{e^{-\alpha z} a_r^2}{2\theta_0^2 z^2}$ and $\gamma_0 := \frac{\theta_0^2}{\sigma_0^2}$.

- ▶ When $\gamma_0 \gg 1$, \mathbb{h}_0 will be close to B_0 with large probability. When $\gamma_1 \gg 1$, \mathbb{h}_1 will be close to B_1 with large probability.

SLIPT Decode-and-Forward UAV Relay



What is the optimum values of δ ?

End-to-End Channel Capacity

- The “weakest link” determines the end-to-end capacity:

$$C(h_0, h_1) = \min(C_0(h_0), C_1(h_0, h_1)). \quad (12)$$

-
-

$$C_0(h_0) = \ln \left(1 + \frac{(\delta P_t h_0)^2}{\sigma_n^2} \right) \quad (13)$$

$$C_1(h_0, h_1) = \ln \left(1 + \frac{((1 - \delta) P_t h_0 h_1)^2}{\sigma_n^2} \right) \quad (14)$$

- The average end-to-end capacity is:

$$\begin{aligned} C &= \int_0^{B_0} \int_0^{B_1} C(h_0, h_1) f_{h_0}(h_0) f_{h_1}(h_1) dh_1 dh_0. \\ &= \int_0^{B_0} \int_0^{B_1} \min \left(\ln \left(1 + \frac{(\delta P_t h_0)^2}{\sigma_n^2} \right), \ln \left(1 + \frac{((1 - \delta) P_t h_0 h_1)^2}{\sigma_n^2} \right) \right) \Phi_1 h_1^{\gamma_1 - 1} \Phi_0 h_0^{\gamma_0 - 1} dh_1 dh_0. \end{aligned} \quad (15)$$

A Closed-Form Expression of Channel Capacity

- ▶ For numerical optimization, we want to obtain a closed-form expression of channel capacity.
- ▶ At some h_1^* , we have that

$$\begin{aligned} \ln \left(1 + \frac{(\delta P_t h_0)^2}{\sigma_n^2} \right) &= \ln \left(1 + \frac{((1-\delta)P_t h_0 h_1^*)^2}{\sigma_n^2} \right) \\ \implies \delta^2 &= (1-\delta)^2 (h_1^*)^2 \implies h_1^* = \frac{\delta}{(1-\delta)}. \end{aligned} \tag{16}$$

- ▶ The channel capacity (after getting rid of the min function) is

$$\begin{aligned} C &= \int_0^{B_0} \int_0^{\min(h_1^*, B_1)} \ln \left(1 + \frac{((1-\delta)P_t h_0 h_1)^2}{\sigma_n^2} \right) \Phi_1 h_1^{\gamma_1 - 1} \Phi_0 h_0^{\gamma_0 - 1} dh_1 dh_0 \\ &\quad + \int_0^{B_0} \int_{\min(h_1^*, B_1)}^{B_1} \ln \left(1 + \frac{(\delta P_t h_0)^2}{\sigma_n^2} \right) \Phi_1 h_1^{\gamma_1 - 1} \Phi_0 h_0^{\gamma_0 - 1} dh_1 dh_0. \end{aligned} \tag{17}$$

A Closed-Form Expression of Channel Capacity

- ▶ **First Approximation:** Replace the log function in (17) with a more tractable function.

$$\ln(x) \approx \alpha \left(x^{\frac{1}{\alpha}} - 1 \right), \quad (18)$$

for large α .

- ▶ **Proof:** Via Taylor series expansion

$$x^{\frac{1}{\alpha}} = 1 + \frac{1}{1!} \frac{1}{\alpha} (x-1) + \frac{1}{2!} \frac{1}{\alpha} \left(\frac{1}{\alpha} - 1 \right) (x-1)^2 + \dots \quad (19)$$

$$\alpha \left(x^{\frac{1}{\alpha}} - 1 \right) = (x-1) + \frac{1}{2!} \left(\frac{1}{\alpha} - 1 \right) (x-1)^2 + \dots$$

$$\lim_{\alpha \rightarrow \infty} \alpha \left(x^{\frac{1}{\alpha}} - 1 \right) = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \dots = \ln(x). \quad (20)$$

Closed-Form Expression of Channel Capacity

- After approximating the log function, the capacity is approximated as

$$\begin{aligned} \mathcal{C} \approx & \int_0^{B_0} \int_0^{\min(h_1^*, B_1)} \left(\alpha \left(1 + \frac{((1-\delta)P_t h_0 h_1)^2}{\sigma_n^2} \right)^{\frac{1}{\alpha}} - \alpha \right) \Phi_1 h_1^{\gamma_1-1} \Phi_0 h_0^{\gamma_0-1} dh_1 dh_0 \\ & + \int_0^{B_0} \int_{\min(h_1^*, B_1)}^{B_1} \left(\alpha \left(1 + \frac{(\delta P_t h_0)^2}{\sigma_n^2} \right)^{\frac{1}{\alpha}} - \alpha \right) \Phi_1 h_1^{\gamma_1-1} \Phi_0 h_0^{\gamma_0-1} dh_1 dh_0. \end{aligned} \quad (21)$$

- Second Approximation:** Through the *binomial approximation*, the quantity

$$\left(1 + \frac{((1-\delta)P_t h_0 h_1)^2}{\sigma_n^2} \right)^{\frac{1}{\alpha}} \approx 1 + \frac{1}{\alpha} \frac{((1-\delta)P_t h_0 h_1)^2}{\sigma_n^2} \quad (22)$$

when $\frac{((1-\delta)P_t h_0 h_1)^2}{\sigma_n^2} \leq 1$ and $\frac{1}{\alpha} \frac{((1-\delta)P_t h_0 h_1)^2}{\sigma_n^2} \ll 1$. The condition $\frac{((1-\delta)P_t h_0 h_1)^2}{\sigma_n^2} \leq 1 \implies h_1 \leq \frac{\sigma_n}{(1-\delta)P_t h_0}$.

Closed-Form Expression of Channel Capacity

- ▶ **Third approximation:** We define $h_1^* := \frac{\sigma_n}{(1-\delta)P_t h_0} \approx \frac{\sigma_n}{(1-\delta)P_t B_0}$ where the approximation holds loosely for $1 < \gamma_0 < 2$, and the approximation gets better when $\gamma_0 \gg 1$.
- ▶ Thus, we have that

$$\left(1 + \frac{((1-\delta)P_t h_0 h_1)^2}{\sigma_n^2}\right)^{\frac{1}{\alpha}} \approx 1 + \frac{1}{\alpha} \frac{((1-\delta)P_t h_0 h_1)^2}{\sigma_n^2}, \quad 0 < h_1 < h_1^* \quad (23)$$

$$\begin{aligned} \left(1 + \frac{((1-\delta)P_t h_0 h_1)^2}{\sigma_n^2}\right)^{\frac{1}{\alpha}} &\approx \left(\frac{((1-\delta)P_t h_0 h_1)^2}{\sigma_n^2}\right)^{\frac{1}{\alpha}} \\ &+ \frac{1}{\alpha} \left(\frac{((1-\delta)P_t h_0 h_1)^2}{\sigma_n^2}\right)^{\frac{1}{\alpha}-1}, \quad h_1 > h_1^* \end{aligned} \quad (24)$$

and

$$\left(1 + \frac{(\delta P_t h_0)^2}{\sigma_n^2}\right)^{\frac{1}{\alpha}} \approx 1 + \frac{1}{\alpha} \frac{(\delta P_t h_0)^2}{\sigma_n^2}, \quad 0 < h_0 < h_0^* \quad (25)$$

$$\left(1 + \frac{(\delta P_t h_0)^2}{\sigma_n^2}\right)^{\frac{1}{\alpha}} \approx \left(\frac{(\delta P_t h_0)^2}{\sigma_n^2}\right)^{\frac{1}{\alpha}} + \frac{1}{\alpha} \left(\frac{(\delta P_t h_0)^2}{\sigma_n^2}\right)^{\frac{1}{\alpha}-1}, \quad h_0 > h_0^* \quad (26)$$

where $h_0^* := \frac{\sigma_n}{\delta P_t}$.

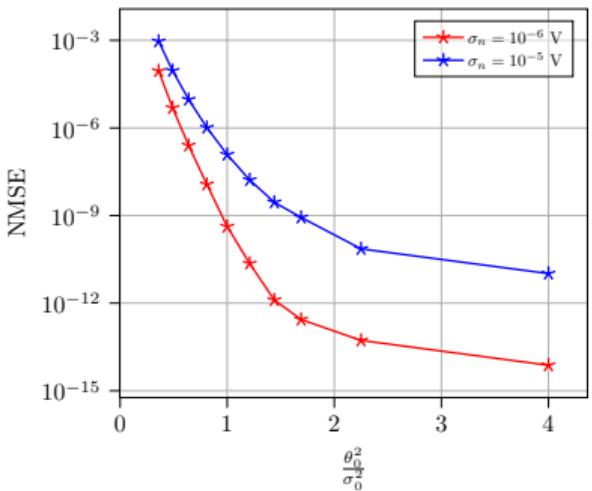
End-to-End Channel Capacity

After application of binomial approximation:

$$\begin{aligned}
 \mathcal{C} \approx \tilde{\mathcal{C}} = & \int_0^{B_0} \int_0^{\min(h_1^*, h_1^*, B_1)} \frac{((1-\delta)P_t h_0 h_1)^2}{\sigma_n^2} \Phi_1 h_1^{\gamma_1-1} \Phi_0 h_0^{\gamma_0-1} dh_1 dh_0 \\
 & + \int_0^{B_0} \int_{\min(h_1^*, h_1^*, B_1)}^{\min(h_1^*, B_1)} \left(\alpha \left(\frac{((1-\delta)P_t h_0 h_1)^2}{\sigma_n^2} \right)^{\frac{1}{\alpha}} + \left(\frac{((1-\delta)P_t h_0 h_1)^2}{\sigma_n^2} \right)^{\frac{1}{\alpha}-1} - \alpha \right) \\
 & \times \Phi_1 h_1^{\gamma_1-1} \Phi_0 h_0^{\gamma_0-1} dh_1 dh_0 \\
 & + \int_{\min(h_1^*, B_1)}^{B_1} \Phi_1 h_1^{\gamma_1-1} dh_1 \left(\int_0^{\min(h_0^*, B_0)} \left(\frac{(\delta P_t h_0)^2}{\sigma_n^2} \right) \Phi_0 h_0^{\gamma_0-1} dh_0 \right. \\
 & \left. + \int_{\min(h_0^*, B_0)}^{B_0} \left(\alpha \left(\frac{(\delta P_t h_0)^2}{\sigma_n^2} \right)^{\frac{1}{\alpha}} + \left(\frac{(\delta P_t h_0)^2}{\sigma_n^2} \right)^{\frac{1}{\alpha}-1} - \alpha \right) \Phi_0 h_0^{\gamma_0-1} dh_0 \right). \tag{27}
 \end{aligned}$$

Approximation Error

$$\text{NMSE} := \frac{\int_0^1 [\mathcal{C}(\delta) - \tilde{\mathcal{C}}(\delta)]^2 d\delta}{\int_0^1 \mathcal{C}(\delta)^2 d\delta}. \quad (28)$$



This figure shows the normalized mean-square error as a function of the factor θ_0^2/σ_0^2 for two values of thermal noise standard deviation σ_n . The total link distance $D = 4000$ m, transmitted power $P_t = 50$ W, the beamwidth θ_1 and angular error standard deviation σ_1 in Channel 1 are 2 mrad and 1 mrad, respectively. The distance $z = 2000$ m, the factor $\beta = 0$, the aperture radius for relay a_r and the destination a_d are 0.2 m and 1 m, respectively.

Closed-Form Optimization Solution Through Approximations

►

$$\underset{\delta}{\text{maximize}} \quad C \\ \text{subject to } 0 < \delta < 1. \quad (29)$$

- $C = \int_0^{B_0} \int_0^{B_1} \min \left(\ln \left(1 + \frac{(\delta P_t h_0)^2}{\sigma_n^2} \right), \ln \left(1 + \frac{((1-\delta)P_t h_0 h_1)^2}{\sigma_n^2} \right) \right) \Phi_1 h_1^{\gamma_1 - 1} \Phi_0 h_0^{\gamma_0 - 1} dh_1 dh_0.$
- For fixed h_0, h_1 , the optimum δ will maximize the integrand \Rightarrow make the arguments of min function equal to each other:

$$\ln \left(1 + \frac{(\delta P_t h_0)^2}{\sigma_n^2} \right) = \ln \left(1 + \frac{((1-\delta)P_t h_0 h_1)^2}{\sigma_n^2} \right). \quad (30)$$

- When $\gamma_0 \gg 1, \gamma_1 \gg 1$, h_1 takes on the value B_1 with a high probability. The suboptimal power split factor, $\tilde{\delta}$, in this case is

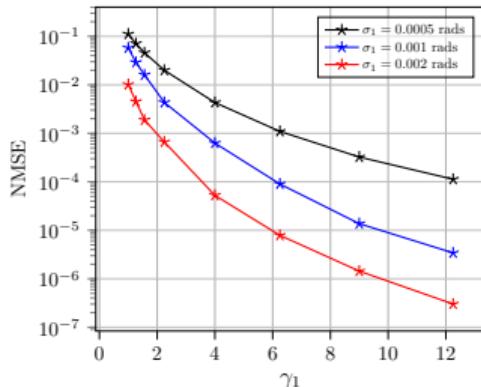
$$\tilde{\delta} := \frac{B_1}{1 + B_1}. \quad (31)$$

- B_1 small implies a poor R-D channel $\Rightarrow 1 - \tilde{\delta} \approx 1 \Rightarrow$ most power should be dedicated to R-D channel.

Maximization of End-to-End Capacity (Continued)

- ▶ The true optimal split factor δ^* is computed through numerical optimization.
- ▶ The normalized mean-square error is

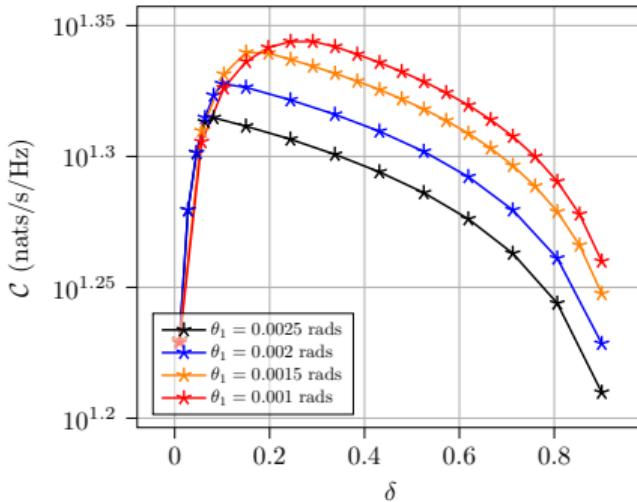
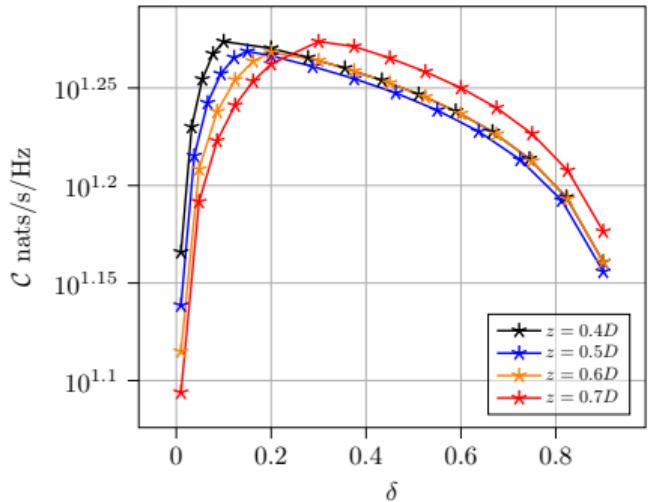
$$\text{NMSE} = \frac{(\delta^* - \tilde{\delta})^2}{(\delta^*)^2}. \quad (32)$$



NMSE as a function of γ_1 for $\gamma_0 = 2$.

- ▶ Typically, $\gamma_0, \gamma_1 = k_0$ where $2 < k_0 < 6$ in practical receivers.

Maximization of End-to-End Capacity (Continued)



End-to-end capacity as a function of power split factor δ for different source-relay distance values and Channel 1 beamwidth θ_1 .

Thank You

muhammad.bashir.1@kaust.edu.sa